

Gammafunktionen $\Gamma(\alpha)$

Definition: $\Gamma(\alpha) = \int_0^{\infty} y^{\alpha-1} e^{-y} dy$

$$\alpha = 1: \quad \Gamma(\alpha) = \int_0^{\infty} e^{-y} dy = \left[-e^{-y} \right]_0^{\infty} = -0 - (-1) = 1$$

Notera att $\int_0^{\infty} e^{-y} dy = 1$

$$\alpha = 2: \quad \Gamma(\alpha) = \int_0^{\infty} ye^{-y} dy = \left[y \cdot (-e^{-y}) \right]_0^{\infty} - \int_0^{\infty} 1 \cdot (-e^{-y}) dy = \left[-\frac{y}{e^y} \right]_0^{\infty} + \int_0^{\infty} e^{-y} dy = 0 - 0 + 1$$

$$g \cdot f = g \cdot F - g' \cdot F$$

Notera att $\int_0^{\infty} ye^{-y} dy = 1$

$$\alpha = 3: \quad \Gamma(\alpha) = \int_0^{\infty} y^2 e^{-y} dy = \left[y^2 \cdot (-e^{-y}) \right]_0^{\infty} - \int_0^{\infty} 2y \cdot (-e^{-y}) dy = \left[-\frac{y^2}{e^y} \right]_0^{\infty} + \int_0^{\infty} 2ye^{-y} dy = 0 - 0 + 2$$

$$g \cdot f = g \cdot F - g' \cdot F$$

Notera att $\int_0^{\infty} y^2 e^{-y} dy = 2$

$$\alpha = 4: \quad \Gamma(\alpha) = \int_0^{\infty} y^3 e^{-y} dy = \left[y^3 \cdot (-e^{-y}) \right]_0^{\infty} - \int_0^{\infty} 3y^2 \cdot (-e^{-y}) dy = \left[-\frac{y^3}{e^y} \right]_0^{\infty} + \int_0^{\infty} 3y^2 e^{-y} dy = 0 - 0 + 6$$

$$g \cdot f = g \cdot F - g' \cdot F$$

Notera att $\int_0^{\infty} y^3 e^{-y} dy = 6$